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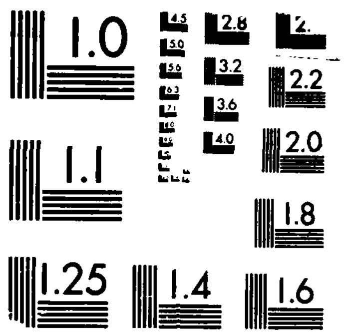

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CMS Technical Summary Report #88-7

FEEDBACK STABILIZATION OF $\frac{du}{dt} = Au + Bf$ IN HILBERT SPACE WHEN $\|f\| < r$

the Normalization Function is < or = r

Marshall Slemrod

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FEEDBACK STABILITIZATION OF $\frac{du}{dt} = Au + Bf$ IN HILBERT SPACE WITH $\|f\| \leq r$

Marshall Slemrod[1,2]

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ABSTRACT

This paper derives a feedback control $f(t)$, $\|f(t)\|_E \leq r$, $r > 0$, which forces the infinite dimensional control system $\frac{du}{dt} = Au + Bf$, $u(0) = u_0 \in H$ to have the asymptotic behavior $u(t) \to 0$ as $t \to \infty$ in H. Here A is the infinitesimal generator of a C_0 semigroup of contractions e^{At} on a real Hilbert space H and B is a bounded linear operator mapping a Hilbert space of controls E into H. An application to the boundary feedback control of a vibrating beam is provided in detail and an application to the stabilization of the NASA Spacecraft Control Laboratory is sketched.

AMS (MOS) Subject Classication: 93D15

Key Words: infinite dimensional control system, feedback stabilization.

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FEEDBACK STABILIZATION OF $\frac{du}{dt} = Au + Bf$ IN HILBERT SPACE WHEN $\|f\| \le r$

Marshall Slemrod[1,2]

1. Introduction

~~In~~ this paper ~~we~~ considers the feedback stablization of a linear control system in an infinite dimensional state space. However unlike the standard feedback control problem where the goal is to find a linear feedback control law, we restrict ourselves to the case where the controls $f(t)$ satisfy ~~the~~ a certain a priori constraint, $\|f(t)\|_E \le r$, $r > 0$. (Here the controls $f(t)$ lie in a Hilbert space E and $\|\cdot\|_E$ denotes the norm of E.) This contraint necessitates a choice of a nonlinear feedback law which drives our state $u(t)$ to zero as $t \to \infty$.

~~We will~~ The author derives such a nonlinear feedback law based on "energy" stability methods. The analysis of the asymptotic behavior of the state $u(t)$ is based on the theories of nonlinear evolution equations and contraction semigroups. While an earlier paper ~~[1]~~ treated a related problem of sub-optimal control the results given here on feedback stabilization are new. A related optimal control problem was considered by Barbu ~~[2]~~.

A strong motivation for this paper has been the work of Hubbard and his co-workers [3], [4] and Balakrishnan [5], [6]. In [3], [4] both laboratory

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experiments and computer simulations for the boundary feedback control of a vibrating beam were described. In particular the problem of [3], [4] corresponds to a special case of theory given here. Hence it is natural to apply the abstract results of this paper to the concrete example of Hubbard et al. This is done in the last section where it is shown that the feedback control given here yields a stablizing feedback. Similarly the papers of Balakrishnan [5], [6] provide a mathematical framework for the stabilization problem of the NASA Spacecraft Control Laboratory Experiment (SCOLE). The last section will also sketch the application of the theory given here to that problem.

The paper is divided into seven sections after this one. Section 2 provides a statement of the abstract control problem and a hint at the method of resolution. Section 3 gives some brief preliminary results on the theory of dynamical systems and nonlinear semigroups. Section 4 uses the ideas of Section 3 to exposite a theorem of Ball and Slemrod on the asymptotic behavior of a class of nonlinear evolution equations. Section 5 applies this theorem to yield one resolution of the feedback stabilization problem (Theorem 5.1). Section 6 gives a survey of the results on asymptotic behavior of nonlinear contraction semigroups and a useful theorem of Dafermos and Slemrod is presented. Section 7 applies Dafermos and Slemrod's result to the feedback stabilization problem in the case A has compact resolvent and $E = \mathbf{R}$ (Theorem 7.1). Section 8 uses the earlier mentioned problems of Hubbard et al and Balakrishnan as illustrative examples.

We note that a good reference for the ideas on nonlinear semigroups and asymptotic behavior is the monograph of A. Haraux [7]. Many of the propositions used here may be found there. In addition numerous examples illustrating the nonlinear semigroup theory are contained there as well.

2. The control problem

Let H be a real Hilbert space with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$. and let E be a second real Hilbert space with inner product $(\cdot,\cdot)$ and norm $\|\cdot\|_E$. Also let A be the infinitesimal generator of a linear C_0 semigroup of contractions on H denoted by e^{At}. (In particular we know $\|e^{At}u_0\| < \|u_0\|$ for all $u_0 \in H$ and $\langle A\phi,\phi\rangle < 0$ for all $\phi \in D(A)$.) Finally let B be a bounded linear operator from E to H.

We consider the abstract control system

$$\frac{du}{dt} = Au + Bf \quad , \tag{2.1}$$

$$u(0) = u_0 \in H \quad . \tag{2.2}$$

Our goal is to find a feedback control

$$f(t) = K(u(t)) \tag{2.3}$$

satisfying the constraint

$$\|f\|_E < r, \; r > 0 \quad , \tag{2.4}$$

which will yield $u = 0$ globally asymptotically stable in some sense.

To do this first formally compute the time rate of change of the "energy":

$$\begin{aligned} \frac{1}{2}\frac{d}{dt}\|u(t)\|_H^2 &= \langle Au,u\rangle + \langle Bf,u\rangle \\ &< (f,B^*u) \quad . \end{aligned} \tag{2.5}$$

Here we have used the fact $\langle Au,u\rangle < 0$. In order to force energy decay yet satisfy the constraint (2.3) we use a saturating control law as suggested in the work of Gutman [8] for finite dimensional systems, i.e. we set

$$\begin{aligned} K(u) &= -\frac{rB^*u}{\|B^*u\|_E} \quad \text{if} \quad \|B^*u\|_E > r \\ &= -B^*u \qquad \text{if} \quad \|B^*u\|_E < r \quad . \end{aligned} \tag{2.6}$$

Notice $K(u)$ is continuous as a function of u and that the desired energy dissipation is obtained:

$$\frac{1}{2}\frac{d}{dt}\|u(t)\|^2 \leq -r\|B^*u\|_E \quad \text{if} \quad \|B^*u\|_E \geq r$$
$$\leq -\|B^*u\|_E^2 \quad \text{if} \quad \|B^*u\|_E < r \; . \tag{2.7}$$

In the next section we will analyze (2.1) with feedback $f(t) = K(u(t))$ and give sufficient conditions for its successful implementation.

3. Preliminary results on nonlinear semigroups

Definitions. Let H be a real Hilbert space. A (generally nonlinear) semigroup $T(t)$ on H is a family of continuous maps $T(t) : H \to H$, $t \in R^+$, satisfying (i) $T(0)$ = identity, (ii) $T(t+s) = T(t)T(s)$, for all $t, s \in R^+$. If in addition $\|T(t)\phi - T(t)\psi\| \leq \|\phi-\psi\|$ for all $\phi, \psi \in H$, $t \geq 0$, $T(t)$ is called a contraction semigroup.

For $\phi \in H$ define the positive orbit through ϕ by $\mathcal{O}^+(\phi) = \bigcup_{t\in R^+} T(t)\phi$. The ω-limit set of ϕ is the (possibly empty) set $\omega(\psi) = \{\psi \in H;$ there exists a sequence $t_n \to \infty$ as $n \to \infty$ such that $T(t_n)\phi \to \psi$ as $n \to \infty\}$. The weak ω-limit set of ϕ is the (possibly empty) set given by $\omega_w(\phi) = \{\psi \in H;$ there exists a sequence $t_n \to \infty$ as $n \to \infty$ such that $T(t_n)\phi \rightharpoonup \psi$ as $n \to \infty\}$. (Here the symbol $\rightharpoonup$ denotes weak convergence in H.)

As subset C of H is said to be positively invariant if $T(t)C \subset C$ for all $t \in R^+$, and invariant if $T(t)C = C$ for all $t \in R^+$.

Theorem 3.1. (i) If $\mathcal{O}(\phi)$ is precompact then $\omega(\phi)$ is a nonempty, invariant set in H. (ii) If each $T(t)$ is sequentially weakly continuous on H (i.e. $T(t)\phi_n \rightharpoonup T(t)\phi$ is $\phi_n \rightharpoonup \phi$), then $\mathcal{O}^+(\phi)$ bounded implies $\omega_w(\phi)$ is a nonempty, invariant set in H.

Proof. (i) The proof is a direct consequence of Prop. 2.2 in Dafermos [9]. (ii) Since $\mathcal{O}^+(\phi)$ belongs to a sequentially weakly compact set in H, $\omega_w(\phi)$ is non-empty. Furthermore, since H is separable this weakly compact set may be regarded as a compact set in a metric space induced by the weak topology (see Dunford and Schwartz [10]). The result again follows from Prop. 2.2 in Dafermos [9].

Hidden in Theorem 3.1 is the essence of this paper. Namely that in the study of the feedback control system described in Section 2 we may need to use, under different circumstances, either part (i) or part (ii) of Theorem 3.1. Roughly the idea is that in the study of nonlinear semigroups of "parabolic" type and nonlinear contraction semigroups of "hyperbolic" type sufficient conditions have been given for $\mathcal{O}^+(\phi)$ to be precompact and hence $\omega(\psi)$ to be nonempty (see Henry [11], Pazy [12], and Dafermos and Slemrod [13]). On the other hand other applications may yield only the information that $\mathcal{O}^+(\phi)$ is bounded and hence the main tool in studying the asymptotic behavior of the feedback system will be the weak ω-limit set.

4. Semilinear evolution equations

We recall some standard results on nonlinear evolution equations. Consider the initial value problem

$$\begin{aligned} \frac{du}{dt} &= Au(t) + F(u(t),t) , \\ u(t_0) &= u_0 , \end{aligned} \tag{4.1}$$

where A is the infinitesimal generator of a linear C_0 semigroup e^{At} on a real Hilbert space H with inner product $\langle\cdot,\cdot\rangle$ and norm $\| \ \|$, $F : H \times \mathbb{R} \to H$ is a given function and $u_0 \in H$ is a given initial datum.

Definition. Let $t_1 > t_0$. A function $u \in C([t_0,t_1];H)$ is a weak solution of (4.1) on $[t_0,t_1]$ if $u(t_0) = u_0$, $F(u(\cdot),\cdot) \in L^1(t_0,t_1;H)$ and if for each $w \in D(A^*)$ the function $\langle u(t),w\rangle$ is absolutely continuous on $[t_0,t_1]$ and satisfies

$$\frac{d}{dt}\langle u(t),w\rangle = \langle u(t),A^*w\rangle + \langle F(u(t),t),w\rangle$$

for almost all $t \in [t_0,t_1]$.

Theorem 4.1. (cf. Balakrishnan [14], Ball [15]). Let $t_1 > t_0$. A function $u : [t_0,t_1] \to H$ is a weak solution of (4.1) if and only if $F(u(t),\cdot) \in L^1(t_0,t_1;H)$ and u satisfies the variation of constants formula

$$u(t) = e^{A(t-t_0)} u_0 + \int_{t_0}^{t} e^{A(t-s)}F(u(s),s)ds$$

for all $t \in [t_0,t_1]$.

The next result characterizes the asymptotic behavior of solutions to (4.1) in an important special case. Also we assume system (3.1) is autonomous, i.e. $F(t,u) = F(u)$, $t_0 = 0$.

Theorem 4.2. Let A generate a linear C_0 semigroup e^{At} of contractions. Let $F : H \to H$ satisfy

(i) F is locally Lipschitz

(ii) $\psi_n \rightharpoonup \psi \Longrightarrow F(\psi_n) \to F(\psi)$,

(iii) $\langle F(\psi),\psi\rangle \leq 0$ for all $\psi \in H$.

Then (4.1) possesses a unique weak solution $u(t;u_0)$ on R^+ for each $u_0 \in H$. Furthermore $T(t)u_0 = u(t,u_0)$ defines a semigroup on H, $\omega_w(u_0)$ is a nonempty invariant set for each $u_0 \in H$, and for each $\psi \in \omega_w(u_0)$

$$\langle T(t)\psi, F(T(t)\psi\rangle = 0 \text{ for all } t \in R^+ .$$

If in addition, the only solution to the above equation is $\psi = 0$, then $u(t;u_0) \rightharpoonup 0$ as $t \to \infty$.

Proof. The proof is given in the paper of Ball and Slemrod [16]. A central idea of the proof is the dissipative mechanism (iii) yields $\mathcal{O}^+(u_0)$ bounded and hence by Theorem 3.1 (ii) $\omega_w(u_0)$ is a nonempty, invariant set in H. The Liapunov functional $\|u(t)\|^2$ is then used to identify $\omega_w(u_0)$ as noted in the theorem.

5. Application of Theorem 4.2 to the stabilization problem

In this section we will discuss the asymptotic behavior of the feedback system (2.1), (2.2), (2.5), (2.6), i.e. we will study the nonlinear evolutionary system

$$\frac{du}{dt} = Au + G(u) \quad , \tag{5.1}$$

$$u(0) = u_0 \quad , \tag{5.2}$$

with

$$\begin{aligned} G(u) &= -\frac{rBB^*u}{\|B^*u\|_E} \quad \text{if} \quad \|B^*u\| > r \quad , \\ &= -BB^*u \qquad \text{if} \quad \|B^*u\| < r \quad , \end{aligned} \tag{5.3}$$

where A, B are as given in Section 2.

Theorem 5.1. For each $u_0 \in H$ there is a unique weak solution $u(t;u_0) = T(t)u_0$ of (5.1), (5.2) defined for all $t \in \mathbb{R}^+$ with $\{0\}$ a stable equilibrium. If in addition B is compact and the only solution of the equation

$$B^* e^{At} \psi = 0 \quad \text{for all} \quad t \in \mathbb{R}^+ \tag{5.4}$$

is $\psi \equiv 0$, then $u(t;u_0) \rightharpoonup 0$ as $t \to \infty$ for all $u_0 \in H$.

Proof. First we note $G(u)$ is globally Lipschitz continuous. For if u_1, $u_2 \in H$ and we set $y_1 = B^* u_1$, $y_2 = B^* u_2$ then $\|y_1\| < r$ and $\|y_2\| > r$ implies

$$
\begin{aligned}
\|G(u_1)-G(u_2)\| &< |||B|||\ \Big\|y_1 - \frac{ry_2}{\|y_2\|_E}\Big\|_E \\
&< \frac{|||B|||}{r}\ \|y_1\|y_2\|_E - ry_2\|_E \\
&< \frac{|||B|||}{r}\ \|y_1(\|y_2\|_E - \|y_1\|_E) + y_1\|y_1\|_E - ry_2\|_E \\
&< \frac{|||B|||}{r}\ (\|y_1\|_E\ \|y_2-y_1\|_E + \|y_1(\|y_1\|_E-r) + r(y_1-y_2)\|_E \\
&< |||B|||(\|y_2-y_1\|_E + \big|\|y_1\| - r\big| + \|y_2-y_1\|) \\
&< 3|||B|||\ \|y_2-y_1\| \ .
\end{aligned}
$$

Here we have used $|||\cdot|||$ to denote the operator norm on the space of bounded linear operators $E \to H$. On the other hand when both y_1, y_2 have norm greater than or equal to r

$$
\begin{aligned}
\|G(u_1)-G(u_2)\| &< |||B|||r\ \Big\|\frac{y_1}{\|y_1\|_E} - \frac{y_2}{\|y_2\|_E}\Big\|_E \\
&< \frac{|||B|||r}{\|y_1\|_E\|y_2\|_E}\ \|y_1\|y_2\|_E - y_2\|y_1\|_E\|_E \\
&< \frac{|||B|||}{r}\ \|y_1\|y_2\|_E - y_1\|y_1\|_E + y_1\|y_1\|_E - y_2\|y_1\|_E\|_E \\
&< \frac{|||B|||}{r}\ (2\|y_1\|_E\ \|y_2-y_1\|_E) \\
&< 2|||B|||\ \|y_2-y_1\|_E \ .
\end{aligned}
$$

Of course if $\|y_1\| < r$, $\|y_2\| < r$ then $\|G(u_1) - G(u_2)\| < |||B|||\ \|y_1-y_2\|_E$ so in general $\|G(u_1) - G(u_2)\| < 3|||B|||^2\|u_1-u_2\|$ for all u_1, $u_2 \in H$.

Next note that the compactness of B implies $G(\psi_n) \to G(\psi)$ if $\psi_n \rightharpoonup \psi$. Also $\langle G(\psi),\psi\rangle = -r\|B^*\psi\|_E$ if $\|B^*\psi\|>r$ and $\langle G(\psi),\psi\rangle = -\|B^*\psi\|_E^2$ if $\|B^*\psi\| < r$.

So Theorem 4.2 applies and tells us that for each $u_0 \in H$ $\omega_w(u_0)$ is a nonempty invariant set in H and for each $\psi \in \omega_w(u_0)$

$$\langle T(t)\psi, G(T(t)\psi)\rangle = 0 \quad \text{for all} \quad t \in \mathbf{R}^+ \quad . \tag{5.5}$$

But $\langle T(t)\psi, G(T(t)\psi\rangle = 0$ for all $t \in \mathbf{R}^+$ implies $B^*T(t)\psi = 0$ for all $t \in \mathbf{R}^+$ which in turn tells us $G(T(t)\psi) = 0$ for all $t \in \mathbf{R}^+$. But then the variation of constants formula of Theorem 4.1 shows for $\psi \in \omega_w(u_0)$ that $T(t)\psi = e^{At}\psi$. So (5.5) in fact implies (5.4) and hence $\omega_w(u_0) = \{0\}$. The fact that $\{0\}$ is stable trivially follows from the estimate $\|T(t)u_0\| \leq \|u_0\|$.

<u>Corollary to Theorem 5.1</u>. Consider the semilinear control system

$$\begin{aligned} \frac{du}{dt} &= Au + Q(u) + Bf \quad , \\ u(0) &= u_0 \quad , \end{aligned} \tag{5.6}$$

where A, B are as in Theorem 5.1 and $Q : H \to H$ is nonlinear, locally Lipschitzian, and dissipative i.e. $\langle Q(u), u\rangle \leq 0$ for all $u \in H$. Then the conclusion of Theorem 5.1 still holds where $f(t) = K(u(t))$.

<u>Proof</u>. If we insert $f(t) = K(u(t))$ into (5.6) our feedback system is

$$\frac{du}{dt} = Au + Q(u) + G(u) \quad .$$

But now that argument given in the proof of Theorem 5.1 applies verbatum with G replaced by $Q + G$.

6. <u>Asymptotic behavior of nonlinear contraction semigroups</u>.

Let $T(t)$ be a nonlinear semigroup of contractions on a real Hilbert space H. We denote $D(\mathcal{A})$ be the set of those $\phi \in H$ for which

$$\lim_{h \to 0+} \frac{T(h)\psi - \psi}{h}$$

exists and define

$$-A\phi = \lim_{h\to 0+} \frac{T(h)\psi-\psi}{h} .$$

It is well known from the theory of nonlinear contraction semigroups (e.g. [17], [7]) that associated with a nonlinear semigroup of contractions there is a unique (possible multi-valued) operator $-A$, that A is maximal monotone, $D(A)$ is dense in H, range $(\lambda A+I) = H$ for any $\lambda > 0$, and $(\lambda A+I)^{-1}$ is a continuous single-valued function.

On the other hand given a maximal monotone operator A we know for every $u_0 \in D(A)$ there exists one and only one function $u(t) : (0,\infty) \to H$ such that

$$\begin{cases} u(t) \in D(A) , \\ \dfrac{du(t)}{dt} \in L^\infty[(0,\infty);\ H] \text{ with} \\ \qquad \left\|\dfrac{du}{dt}\right\|_{L^\infty[(0,\infty);H]} \le \|A^0 u_0\| , \\ \dfrac{du(t)}{dt} + Au(t) \ni 0 \text{ on } (0,\infty) , \\ u(0) = u_0 . \end{cases}$$

Here A^0, the minimal section of A, is the function which assigns to each $\phi \in D(A)$ that element of $A\phi$ which has least norm. Furthermore u is right - differentiable at any $t \in [0,\infty)$ and

$$\frac{d^+u(t)}{dt} = -A^0u(t)$$

for all $t \in [0,\infty)$.

If u and v are solutions associated to initial data u_0 and v_0 then

$$\|u(t) - v(t)\|_H \le \|u_0-v_0\|_H \quad \text{for } t \ge 0 .$$

We note by $T(t)$ the extension by continuity of the map $u_0 \in D(A) \to u(t) \in D(A)$ to $\overline{D(A)}$. If $D(A)$ is dense in H then $T(t)$ defines the nonlinear contraction semigroup on H "generated" by $-A$.

The asymptotic behavior of nonlinear contraction semigroups has been characterized by the following theory of Dafermos and Slemrod [13].

<u>Theorem 6.1</u>. Let A be a maximal monotone operator on a Hilbert space H. Assume $0 \in$ range (A) and $(\lambda A+I)^{-1}$ is compact for some $\lambda > 0$. Then for any $u_0 \in \overline{D(A)}$ the weak solution of the Cauchy problem

$$\frac{du}{dt} + Au(t) \ni 0$$

$$u(0) = u_0 \quad ,$$

given by $u(t) = T(t)u_0$ approaches as $t \to \infty$ a compact subset Ω of a sphere $\{y;\ \|y-a\| = r\}$, $r < \|u_0-a\|$, $a \in A^{-1}0$. Furthermore Ω is minimal, invariant, and equi-almost periodic under the semigroup $T(t)$ generated by $-A$ and T restricted to the closed convex hull of Ω $\overline{co\Omega}$ is an affine group of isometries. If in addition $u_0 \in D(A)$ then the set Ω is contained in $D(A)$, and $A^0\Omega$ is compact and lies on a sphere centered at 0. Moreover $\overline{co\Omega} \subset D(A)$ and the restriction of A^0 to $\overline{co\Omega}$ is affine.

While the proof is contained in [13] and [7] we note the main idea again relates back to Theorem 3.1. Recall that we noted after the statement of Theorem 3.1 that part (i) of the theorem applied to certain "hyperbolic" semigroups where we could actually show $O^+(\phi)$ is precompact. Theorem 6.1 identifies a class of these semigroups as ones arising from nonlinear contraction semigroups with compact resolvent. The rest of the theorem essentially follows by identifying the ω-limit of $T(t)\phi$ with the aid of the Liapunov functional $\|u(t)\|^2$.

7. Application of Theorem 6.1 to the stabilization problem.

In this section we discuss the application of Theorem 6.1 in the special case $E = \mathbb{R}$. In this case B is a fixed element in H. Furthermore note that for $\phi, \psi \in D(A)$ that $-A-G$ is monotone. To see this set $y = B^*\phi$, $z = B^*z$ and observe

(i) for $\|y\|_E < r$, $\|z\|_E < r$,

$$\langle\phi-\psi, A\phi + G(\phi) - A\psi - G(\psi)\rangle < -|y-z|^2;$$

(ii) for $\|y\|_E > r$, $\|z\|_E > r$,

$$\langle\phi-\psi, A\phi + G(\phi) - A\psi - G(\psi)\rangle < -r(y-\xi)\cdot\left(\frac{y}{|y|} - \frac{\xi}{|\xi|}\right) < 0 ;$$

(iii) for $\|y\|_E < r$, $\|z\|_E > r$,

$$\langle\psi-\psi, A\phi + G(\phi) - A\psi - G(\psi)\rangle < -(y-\xi)(y - \frac{r\xi}{|\xi|})$$

$$< -y^2 + r|y| + |y|\ |\xi| - r|\xi|$$

$$(|y|-r)(|\xi|-|y|) < -(|y|-r)^2.$$

Hence we see that the operator $\mathcal{A} = -A - G$ defined on $D(A)$ is monotone. Notice also that the above argument also shows $-G$ is monotone. Since $-G$ is continuous (see Section 5) and $-A$ is maximal monotone (by the Hille-Yosida-Phillips Theorem) a theorem of G. F. Webb [18] asserts that the sum $-A -G$ is maximal monotone.

Now we are prepared to prove our stabilization theorem.

Theorem 7.1. For each $u_0 \in H$ there exists a unique weak solution of (5.1), (5.2) for all $t \in R^+$ with $\{0\}$ a stable equilibrium of (5.1). If in addition $E = \mathbb{R}$, $(\lambda I-A)^{-1}$ is compact for all real $\lambda > 0$, and the only solution of the equation

$$B^*e^{At}\psi = 0 \quad \text{for all} \quad t \in R^+$$

is $\psi = 0$, then $u(t,u_0) \to 0$ as $t \to \infty$ for all $u_0 \in H$.

Notice here that Theorem 7.1 improves on Theorem 5.1 in that weak convergence is now replaced by strong convergence. Of course the price paid

is that we assume $E = \mathbb{R}$ and A has compact resolvent.

Proof of Theorem 7.1. We shall apply Theorem 6.1. We have already shown $\hat{A} = -A - G$ is maximal monotone and also trivially $0 \in \text{range}(\hat{A})$. Now let $\{h_n\}$ be a bounded sequence in H with $u_n \in D(A)$ such that

$$-Au_n - G(u_n) + \frac{u_n}{\lambda} = \frac{h_n}{\lambda} \tag{7.1}$$

for $\lambda > 0$. Take the inner produce of both sides of (7.1) with u_n to see

$$\|u_n\|^2 \leq \langle u_n, h_n \rangle \leq \|u_n\| \, \|h_n\|$$

and hence $\|u_n\| \leq \|h_n\|$. Thus $\{u_n\}$ also belongs to a bounded set in H. Rewrite (7.1) as

$$-\left(A - \frac{1}{\lambda} I\right)u_n = \frac{h_n}{\lambda} + G(u_n) \quad . \tag{7.2}$$

The Lipschitz continuity of G shows the right hand side of (7.2) lies in a bounded set of H and the compactness of $(A-\lambda I)^{-1}$ for $\lambda > 0$ shows $\{u_n\}$ lies in a compact subset of H. Thus $(\lambda \hat{A} + I)^{-1}$ is compact for all $\lambda > 0$.

Now let $u_0 \in D(A)$. Theorem 6.1 tells us $u(t) = T(t)u_0$, the strong solution of (5.1), (5.2), approaches as $t \to \infty$, a compact subset Ω of a sphere $\{y; \|y\| = r\}$ where $\Omega \subset D(A)$. Let $v_0 \in \Omega$. Since Ω is invariant we must have $\|T(t)v_0\| = r$ for all $t \in \mathbb{R}^+$, and differentiation with respect to t and use of (5.1) shows $B^*T(t)v_0 = 0$ for $t \in \mathbb{R}^+$. But (7.4) coupled with (5.1) and the invariance of Ω shows $T(t)v_0 = e^{At}v_0$ for $v_0 \in \Omega$. Hence for $v_0 \in \Omega$, $B^*e^{At}v_0 = 0$ for all $t \in \mathbb{R}^+$. But by the hypothesis of the theorem $v_0 = 0$ and $\Omega = \{0\}$ for $u_0 \in D(A)$. Since $D(A)$ is dense in H and $T(t)$ is a contraction the triangle inequality readily shows $\Omega = \{0\}$ for all $u_0 \in H$ as well. Since the existence, uniqueness, and stability have already been prove in Theorem 5.1 the proof is complete.

8. Examples

8.1. Boundary feedback control of a vibrating beam.

In [3], [4] Hubbard and his co-workers considered the following boundary control system for a cantilever beam.

Denote by $w(x,t)$ the displacement of the beam where w satisfies

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^4 w}{\partial x^4} = 0 \quad \text{for} \quad 0 < x < L \ ,$$

$$w = \frac{\partial w}{\partial x} = 0 \qquad \text{for} \quad x = 0 \ , \tag{.8.1}$$

$$\left.\begin{aligned} \frac{\partial^2 w}{\partial x^2} &= - \frac{\partial^3 w}{\partial t^2 \partial x} + f(t) \\ \frac{\partial^3 w}{\partial x^3} &= \frac{\partial^2 w}{\partial t^2} \end{aligned}\right\} \qquad \text{for} \quad x = L \ .$$

Here $w(x,t)$ denotes the displacement of a beam and $f(t)$ is an applied scalar boundary control, $|f(t)| < r$.

In addition we prescribe initial conditions on the displacement and velocity of the beam,

$$\begin{aligned} w(x,0) &= w_0(x) \ , \\ w_t(x,0) &= v_0(x) \ , \qquad 0 < x < L \ . \end{aligned} \tag{8.2}$$

For analytical convenience we rewrite (8.1) in the following first order form

$$\begin{aligned} \frac{\partial w}{\partial t} &= v \\ \frac{\partial v}{\partial t} &= - \frac{\partial^4 w}{\partial x^4} \\ \frac{da}{dt} &= \left.\frac{\partial^3 w}{\partial x^3}\right|_{x = L} \\ \frac{db}{dt} &= \left.\frac{\partial^2 w}{\partial x^2}\right|_{x = L} + f(t) \end{aligned} \tag{8.3}$$

where we require $a(t) = \frac{\partial w}{\partial t}\Big|_{x = L}$, $b(t) = \frac{\partial^2 w}{\partial t \partial x}\Big|_{x = L}$.

The above first order formulation motivates us to formally define the linear operator A

$$A\begin{bmatrix} w \\ v \\ a \\ b \end{bmatrix} = \begin{bmatrix} v \\ -\dfrac{d^4 w}{dx^4} \\ \dfrac{d^3 w}{dx}\Big|_{x = L} \\ \dfrac{d^2 w}{dx^2}\Big|_{x = L} \end{bmatrix}$$

so that when

$$u = \begin{bmatrix} w \\ v \\ a \\ b \end{bmatrix} \quad , \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

(5.3) has the form

$$\frac{du}{dt} = Au + Bf \quad . \tag{8.4}$$

To be precise we must identify the domain of definition of A. To this extent we first define the Hilbert space H:

$$H = \{(w,v,a,b) \in H^2\,(0,1) \times L^2(0,1) \times R \times R \;;$$
$$w = w' = 0 \quad \text{at} \quad x = 0\}$$

endowed with the inner product

$$\langle (w,v,a,b),\ (\tilde{w},\tilde{v},\tilde{a},\tilde{b})\rangle =$$
$$\int_0^L (w''(x)\ \tilde{w}''(x) + v(x)\tilde{v}(x))dx + a\tilde{a} + b\tilde{b} \quad .$$

Now we take the domain of A as

$$D(A) = \{(w,v,a,b) \in H^4(0,1) \times H^2(0,1) \times \mathbf{R} \times \mathbf{R} ;$$

$$w = w' = 0,\ v = v' = 0 \ \text{ at } \ x = 0,\ v = a,\ \frac{dv}{dx} = b \ \text{ at } \ x = L\} \ .$$

It is an easy computation to see that $\langle Au,u\rangle$ is dissipative for $u \in D(A)$. In particular this shows that A is dissipative i.e. $\langle Au,u\rangle \leq 0$ for $u \in D(A)$ or equivalently $-A$ is monotone $\langle -Au,u\rangle \geq 0$ for $u \in D(A)$.

We also note that A is an infinitesimal generator of a linear C_0 semigroup of contractions $T(t)$ on H. To show this we simply apply the Lumer-Phillips Theorem [18] which asserts A will be the generator of such a semigroup if and only if A is dissipative, densely defined on H, and satisfies the range condition $R(-A + \lambda_0 I) = H$ for some $\lambda_0 > 0$.

We have already shown A is dissipative and it is a simple observation to see that it is densely defined. To check the range condition we let (g,h,c,d) be a generic element of H. Then satisfying the range condition is equivalent to finding $u = (w,v,a,b) \in D(A)$ with

$$-v + \lambda_0 w = g \ , \tag{8.5}$$

$$\frac{d^4w}{dx^4} + \lambda_0 v = h \ , \tag{8.6}$$

$$\left.\frac{d^3w}{dx^3}\right|_{x = L} + \lambda_0 a = c \ , \tag{8.7}$$

$$\left.\frac{d^2w}{dx^2}\right|_{x = L} + \lambda_0 b = d \ . \tag{8.8}$$

From the first two of these equations we see w should satisfy

$$\frac{d^4w}{dx^4} + \lambda_0^2 w = h + \lambda_0 g \ . \tag{8.9}$$

We now solve the ordinary differential equation subject to the boundary conditions $w(0) = 0$, $w'(0) = 0$, and

$$-\frac{d^3w}{dx^3} + \lambda_0^2 w = c + \lambda_0 g \quad , \tag{8.10}$$

$$\frac{d^2w}{dx^2} + \lambda_0^2 \frac{dw}{dx} = d + \lambda_0 \frac{dg}{dx} \tag{8.11}$$

at $x = L$. This can be done by explicitly solving (5.9) with four constants of integration and then using the four boundary conditions to evaluate the constants. This will yield $w \in H^4(0,1)$ with $w(0) = w'(0) = 0$, which with v defined by (5.5) and a, b given by $a = v(L)$, $b = \frac{dv}{dx}(L)$ solve (5.5)-(5.8). Straightforward inspection of (5.5)-(5.8) also shows $u = (w,v,a,b)$ is in $D(A)$.

This analysis works for any λ_0 real. This is no surprise since as we shall prove the spectrum of A is purely imaginary, discrete, of the form $\lambda = \pm i\mu^2$ where μ satisfies the transcendental equation

$$\begin{aligned} &1 + \cos\mu L \cosh\mu L + \mu(\sinh\mu L \cos\mu L - \cosh\mu L \sin\mu L) \\ &- \mu^3(\cos\mu L \sinh\mu L + \sin\mu L \cosh\mu L) + \mu^4(1 - \cos\mu L \cosh\mu L) = 0 \, . \end{aligned} \tag{8.12}$$

Finally we trivially note that B is a bounded linear operator on the control space $E = \mathbb{R}$ to H. Hence we have rewritten the boundary control system (8.1), (8.2) in the form (2.1), (2.2). Here the initial data is

$$u_0 = \begin{bmatrix} w_0 \\ w_1 \\ a_0 \\ b_0 \end{bmatrix} \in H$$

and we set $(a,\tilde{a})_E = a\tilde{a}$, $a,\tilde{a} \in \mathbb{R}$.

Also since the imbedding of $H^4(0,1) \times H^2(0,1) \to H^2(0,1) \times L^2(0,1)$ is compact we see $D(A)$ is compactly imbedded in H. Hence $(A - \lambda_0 I)^{-1}$ is compact for any real λ_0.

So far we have shown A is the infinitesimal generator of a C_0 semigroup of contractions $T(t)$ on H, $(A-\lambda_0 I)^{-1} : H \to H$ is compact. To apply Theorem 7.1 we must now show $B^* e^{At}\psi = 0$, $\psi \in H$, for all $t \in \mathbf{R}^+$, implies $\psi = 0$. But if $B^* e^{At}\psi = 0$ then $\langle Bq, e^{At}\psi\rangle = 0$ for all $q \in \mathbf{R}$. But by the definition of B if we write $\hat{u}(t) = e^{At}\psi = [\hat{w}(t), \hat{v}(t), \hat{a}(t), \hat{b}(t)]$ then $q\hat{b}(t) = 0$ for all $q \in \mathbf{R}$ and $t \in \mathbf{R}^+$, i.e. $\hat{b}(t) = 0$ for all $t \in \mathbf{R}^+$. So our goal now is to show $\hat{b}(t) = 0$ for all $t \in \mathbf{R}^+$ implies $\psi = 0$.

To do this we shall compute $e^{At}\psi$ for $\psi \in D(A)$ explicitly. First let us extend H to the complex Hilbert space $\mathcal{H} = H \oplus iH$ where $\mathcal{H}$ has inner product $\langle\langle\ ,\ \rangle\rangle$

$$\langle\langle x_1 + iy_1, x_2 + iy_2\rangle\rangle =$$

$$\langle x_1, x_2\rangle + \langle y_1, y_2\rangle - i\,[\langle x_1, y_2\rangle - \langle x_2, y_1\rangle] \quad .$$

Then A is also the infinitesimal generator a C_0 semigroup on

$$\mathcal{H} : e^{At}\chi = e^{At}\mathrm{Re}\ \chi + ie^{At}\mathrm{Im}\ \chi \quad \text{for} \quad \chi \in \mathcal{H} \quad .$$

The advantage of introducing $\mathcal{H}$ is that we can now represent $e^{At}u_0$ as an eigenfunction expansion. To this end let $u \in D(A)$ so that $Au = \lambda u$. Then if we write

$$u = \begin{bmatrix} w \\ v \\ a \\ b \end{bmatrix} \in D(A)$$

u must satisfy

$$\begin{aligned} &-v + \lambda w = 0 \\ &w''''(x) + \lambda v = 0 \\ &\left.\begin{aligned} -w''' + \lambda a &= 0 \\ w'' + \lambda b &= 0 \end{aligned}\right\} \quad \text{at} \quad x = L \quad , \\ &w = w' = 0 \quad \text{at} \quad x = 0 \quad , \end{aligned}$$

and $a = \lambda w(L)$, $b = \lambda w'(L)$ since $u \in D(A)$. Doing the obvious eliminations we see w must satisfy

$$w''''(x) + \lambda^2 w(x) = 0, \quad 0 < x < L \quad , \tag{8.13}$$

$$w = w' = 0 \quad \text{at} \quad x = 0 \quad , \tag{8.14}$$

$$\left.\begin{aligned} -w''' + \lambda^2 w &= 0 \\ w'' + \lambda^2 w' &= 0 \end{aligned}\right\} \quad \text{at} \quad x = L \quad . \tag{8.15}$$

From (8.13), (8.14) we know $w(x)$ is of the form $w(x) = C(\sin \mu x - \sinh \mu x) + D(\cos \mu x - \cosh \mu x)$ where $\lambda = \pm i\mu^2$. From (8.15) we seen the additional relations

$$\begin{aligned} &C(\sin \mu L + \sinh \mu L) + D(\cos \mu L + \cosh \mu L) = \\ &\quad -\mu^2\{C(\cos \mu L - \cosh \mu L) + D(-\sin \mu L - \sinh \mu L)\} \end{aligned} \tag{8.16}$$

and

$$\begin{aligned} &C(\cos \mu L + \cosh \mu L) + D(-\sin \mu L + \sinh \mu L) = \\ &\mu\{C(\sin \mu L - \sinh \mu L) + D(\cos \mu L - \cosh \mu L)\} \end{aligned} \tag{8.17}$$

must be satisfied.

Elimination of C and D from (8.16), (8.17) yields the spectral formula (8.12). For each μ satisfying (8.12) the associated C and D are determined from either (8.14) or (8.17). If we denote the positive solutions of (8.12) by $\{\mu_n\}$ then $0 < \mu_1 < \mu_2 < \quad \to +\infty$ and the eigenvalues are $\lambda_n = +i\mu_n^2$, $n = 1,2,\ldots,\lambda_{-n} = -i\mu_n^2$, $n = 1,2,3,\ldots$. The associated eigenfunctions are

$$u_n = \begin{bmatrix} w_n \\ +i\mu_n^2 w_n \\ +i\mu_n^2 w_n(L) \\ +i\mu_n^2 w_n(L) \end{bmatrix} \text{ for } \lambda_n \quad , \quad n = 1,2\ldots$$

and

$$u_{-n} = \begin{bmatrix} w_n \\ -i\mu_n^2 w_n \\ -i\mu_n^2 w_n(L) \\ -i\mu_n^2 w_n(L) \end{bmatrix} \qquad \text{for } \lambda_{-n} \ , \quad n = 1,2\ldots \quad .$$

(Here we have used $v_n = \lambda_n w_n$.)

From (8.13), (8.14), (8.15) we find

$$\int_0^L w_n''(x)w_m''(x)dx + \lambda_{\pm n}^2 \int_0^L w_n(x)w_m(x)dx + \lambda_{\pm n}^2 [w_n w_m + w_n w_m]_{x=L} = 0 \quad . \qquad (8.18)$$

Now interchange m and n in (8.18) and take the difference of the two equations. This will show that

$$\left.\begin{array}{l} \int_0^L w_n(x)w_m(x) + [w_n w_m + w_n w_m]_{x=L} = 0 \ , \\ \int_0^L w_m''(x)w_n''(x)dx = 0 \ , \end{array}\right\} \quad \text{if} \quad m \neq n \ . \qquad (8.19)$$

If we set $m = n$ (8.18) we see as well that

$$\int_0^L w_n''(x)^2 dx = \mu_n^4 \ \left\{\int_0^L w_n^2(x)dx + [w_n^2 + w_n^2]_{x=L}\right\} \quad . \qquad (8.20)$$

From (8.19) we see that $\langle\langle u_n, u_m\rangle\rangle = 0$ if $m \neq n$ and $|m| \neq |n|$ and from (8.20) we see $\langle\langle u_n, u_{-n}\rangle\rangle = 0$ as well, $n = 1,2,\ldots$. Hence $\{u_n\}$, $n = \pm 1, \pm 2, \ldots$ of eigenfunctions forms an orthonomal set in H. It is easy to show it is complete as well. Furthermore we assume it is normalized so that $\langle\langle u_n, u_n\rangle\rangle = 1$. Hence if $u_0 \in D(A) \subset H$

$$\hat{u}(t) = e^{At}u_0 = \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} c_n e^{\lambda_n t} u_n \qquad (8.21)$$

where the $\{c_n\}$ are complex coefficients satisfying

$$c_n = \langle\langle u_0, u_n\rangle\rangle, \ n = \pm 1, \pm 2, \ldots \quad . \qquad (8.22)$$

Since $\overline{u}_n = u_{-n}$, $\overline{\lambda}_n = \lambda_{-n}$, (where the overbar denotes complex conjugation) we can write (8.21) as

$$\hat{u}(t) = \sum_{n=1}^{\infty} c_n e^{\lambda_n t} u_n + \sum_{n=1}^{\infty} c_{-n} \overline{e^{\lambda_n t} u_n} .$$

Because u_0 is real we must have $\overline{c}_n = c_{-n}$ as well and

$$\hat{u}(t) = 2 \operatorname{Re} \sum_{n-1}^{\infty} c_n e^{i\mu_n^2 t} u_n . \tag{8.23}$$

As advertised earlier our condition $B^* e^{At}\psi = 0$ for all $t \in R^+$ is equivalent to the fourth component $\hat{b}(t) = 0$ for all $t \in R^+$. If we write $c_n = \alpha_n + i\beta_n$ and use (8.23) this means for all $t \in R^+$ that

$$\sum_{n-1}^{\infty} (\alpha_n \sin \mu_n^2 t + \beta_n \cos \mu_n^2 t)\, \mu_n^2 w_n'(L) = 0 . \tag{8.24}$$

The left hand side of (8.24) defines an almost periodic function and hence by the uniqueness theorem for almost periodic functions [19] the coefficients mush vanish, i.e.

$$\left.\begin{aligned} \alpha_n \mu_n^2\, w_n'(L) &= 0 , \\ \beta_n \mu_n^2\, w_n'(L) &= 0 , \end{aligned}\right\} \quad n = 1, 2, \ldots . \tag{8.25}$$

From the spectral relation (8.12) we know $\mu_n \neq 0$. Furthermore (8.13)-(8.15) when coupled with $w_n' = 0$ at $x = L$ force $w_n \equiv 0$. But this contradicts the u_n being eigenfunctions so we know $w_n'(L) \neq 0$. Hence (8.25) implies $\alpha_n = \beta_n = 0$, i.e. $\psi = 0$. We can now apply Theorem 7.1 to conclude the following result.

<u>Theorem 8.1</u>. Consider the control system (8.1)-(8.2). Then feedback control

$$\begin{aligned} f(t) &= -r \operatorname{sgn} w_{tx}(L,t) && \text{for } |w_{tx}(L,t)| > r \\ &= -w_{tx}(L,t) && \text{for } |w_{tx}(L,t)| < r \end{aligned}$$

yields a unique weak solution to (8.1), (8.2) for initial data (w_0, v_0, a, b) in $H = \{(w,v,a,b) \in H^2(0,1) \times L^2(0,1) \times R \times R;\ \dot{w} = w' = 0 \text{ if } x = 0\}$. In addition the zero solution of (8.1) is uniformly asymptotically stable in H.

<u>Proof</u>. Observe that $u = (w,v,a,b)$, $B^*u = w_{tx}(L,t)$. So use of Theorem 7.1 and the definition of uniform asymptotic stability gives the result.

Notice that Theorem 5.1 also applies in this case since B is compact. However as noted earlier Theorem 7.1 yields the stronger result in that convergence to the zero equilibrium solution is in the strong rather than weak topology.

8.2. Boundary control of the NASA Spacecraft Laboratory Control Experiment (SCOLE)

An example where Theorem 5.1 applies and Theorem 7.1 does not is provided by the stabilization problem for the NASA Spacecraft Laboratory Control Experiment (SCOLE). Since the details of the mathematical model (which are very lengthy) are provided in the papers of Balakrishnan [5] and Balakrishnan and Taylor [6] we only sketch the main ideas here.

First we quote from [5] for a description of the physical problem.

> "The physical apparatus consists of a softly supported antenna attached to the space shuttle by a flexible beam like truss. The control objective is to slew the antenna on command within the given accuracy and maintain stability, based on noisy sensor data and limited control authority; allowance must be made for random disturbance. The control forces and torques are applied at the shuttle end as well as the antenna end and in addition provision is made for a small number of z-axis proof-mass activators along the beam."

In [5] Balakrishnan shows that in the absense of noise the problem can be reformulated in the form (5.6) where $H = (L_2(0,L))^3 \times \mathbf{R}^{14}$, $\langle Au,u \rangle = 0$, $\langle Q(u),u \rangle = 0$, $E = \mathbf{R}^{10}$, and $B^* e^{At}\psi = 0$ for all $t \in \mathbf{R}^+$ implies $\psi = 0$. So the Corollary to Theorem 5.1 implies the zero solution is stable and furthermore $u(t,u_0) \to 0$ as $t \to \infty$ for all $u_0 \in H$. So the feedback control law $f(t) = K(u(t))$ provides a "weakly" damping resolution of the SCOLE problem under the constraint $f_E < r$. Of course since $E = \mathbf{R}^{10}$ and not $\mathbf{R}$ Theorem 7.1 does not apply.

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